Blind Pilot Decontamination

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("HARP")
Massive MIMO

Massive MIMO mimics the idea of spread spectrum.

- **Spread spectrum:**
  - Massive use of bandwidth
Massive MIMO

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- Spread spectrum:
  - Massive use of \textit{bandwidth}
  - Large processing gain
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- **Massive MIMO:**
  - Massive use of *antenna elements*
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  - Large array gain

*Both systems can operate in arbitrarily strong noise and interference.*
Introduction

Uplink (Reverse Link) System Model

\[ R \gg T \quad L \sim T \]
For $T$ transmit antennas and $R$ receive antennas, even for a static channel, $RT$ channel coefficients must be estimated.

- **Linear channel estimation:**
  - The array gain, can be utilized for data detection, but **not** for channel estimation.
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*How to estimate a massive MIMO channel appropriately?*
We propose an uplink (reverse link)-based approach:

For a reciprocal channel, it suffices to utilize the array gain on the uplink.

Once, we have reliably detected the uplink data, we can use all uplink data to estimate the downlink (forward link) channel to high accuracy.
Blind Interference Rejection

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- Implementation:
  - Project onto the orthogonal complement of the interference subspace.
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Implementation:
- Project onto the orthogonal complement of the interference subspace.

How to find the interference subspace or its orthogonal complement?
Matched Filter Projection

Let us start the considerations with solely white noise and a SIMO system.
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- Let \( y_c \) be the column vector received at the receive array at time \( c \) and \( Y = [y_1, \ldots, y_C] \) with \( C \) denoting the coherence time.

\[
\begin{align*}
&\text{We would like to find a linear filter } m, \text{ such that } m^\dagger Y \text{ has high SNR.} \\
&\text{Then, we find } m = \text{argmax}_{m_0} ||m_0^\dagger Y||^2_2 = \text{argmax}_{m_0} m_0^\dagger YY^\dagger m_0 m_0^\dagger m_0 \\
&\text{is that eigenvector of } YY^\dagger \text{ that corresponds to the largest eigenvalue.}
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- We would like to find a linear filter $\mathbf{m}$, such that $\mathbf{m}^\dagger \mathbf{Y}$ has high SNR.
- Then, we find

$$\mathbf{m} = \arg\max_{\mathbf{m}_0} \frac{||\mathbf{m}_0^\dagger \mathbf{Y}||^2}{||\mathbf{m}_0||^2} = \arg\max_{\mathbf{m}_0} \frac{\mathbf{m}_0^\dagger \mathbf{Y} \mathbf{Y}^\dagger \mathbf{m}_0}{\mathbf{m}_0^\dagger \mathbf{m}_0}$$

is that eigenvector of $\mathbf{Y} \mathbf{Y}^\dagger$ that corresponds to the largest eigenvalue.
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- We now project the received signal onto that subspace

$$\mathbf{Y}' = \mathbf{M}^\dagger \mathbf{Y}$$

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*We have utilized the array gain without estimating the channel.*
Consider now the general case (noise, interference and a MIMO system with $T > 1$ transmit antennas and $R \gg T$ receive antennas).
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$$SNR \gg \frac{T}{R},$$

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Matched Filter Projection III

Consider now the general case (noise, interference and a MIMO system with \( T > 1 \) transmit antennas and \( R \gg T \) receive antennas).

- While white noise is small in all components if
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  \text{SNR} \gg \frac{T}{R},
  \]
  the interference typically concentrates in few signal dimensions where it is strong.

*How to distinguish the signal of interest from interference?*
Consider power-controlled hand-off and perfect received power control.

Interfering signals cannot be stronger than signals of interest, i.e. $P \geq I$.

Most interfering signals are noticeably weaker than the signals of interest. For vanishing load $\alpha = T / R \to 0$, the signals of interest can be separated from the interference.

What if the load is small, but not vanishing?
Power Controlled Hand-Off

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Empirical Eigenvalue Distribution

\[ p_\lambda(\lambda) \]

\[ R = 300 \]
\[ T = 10 \]
\[ C = 1000 \]
\[ L = 2 \]
\[ W = 1000 \]
\[ P = 100 \]
\[ I = 25 \]
Eigenvalue Spread

Assume an i.i.d. channel matrix and \( R \gg T \to \infty \).

The eigenvalues of the signal of interest are confined in an interval centered at the received power \( P \) with width

\[
4P \sqrt{\frac{T}{R} + \frac{T}{C}}.
\]

For massive MIMO, the width is quite small.
**Eigenvalue Spread**

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The eigenvalues of the sole interference spread around the interference power (which for sake of simplicity is assumed to be unique). They are confined in an interval centered at the interference power $I$ with width

$$4I\sqrt{\frac{LT}{R} + \frac{LT}{C}},$$

where $L$ denotes the number of interfering cells.

*For massive MIMO, the width is quite small.*
Eigenvalue Separation

The two intervals do not overlap if

$$\frac{P}{I} > \frac{1 + 2\sqrt{\frac{LT}{R} + \frac{LT}{C}}}{1 - 2\sqrt{\frac{T}{R} + \frac{T}{C}}}.$$
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P \frac{I}{l} > \frac{1 + 2\sqrt{\frac{LT}{R}} + \frac{LT}{C}}{1 - 2\sqrt{\frac{TR}{R} + \frac{TC}{C}}}.\]

If the two intervals do not overlap, we can totally reject the interference by means of eigenvalue decomposition.
Eigenvalue Separation

The two intervals do not overlap if

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*If the two intervals do not overlap, we can totally reject the interference by means of eigenvalue decomposition.*

For finite number of receive antennas, the interval boundaries are *not sharp*, but have exponentially decaying tails.
Simulation Results

BER vs. Array Size

\[ T = 3 \]
\[ C = 1000 \]
\[ L = 2 \]
\[ \text{SNR} = -10 \text{dB} \]

1 pilot symbol per transmit antenna and cell
BER vs. Power Margin

\[ I/P \]

uncoded BER

threshold for no overlap

conv. method of Marzetta

proposed subspace method

\[ R = 200 \]
\[ T = 2 \]
\[ C = 400 \]
\[ L = 2 \]
\[ W = 1 \]
\[ P = 0.1 \]

1 (-) or 10 (- -) pilot symbols per transmit antenna and cell
Power Margin

How to guarantee a sufficient power margin between the signal of interest and the interference?
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If a user experiences equally good channel conditions to several base stations/access points, the user forms a beam that favors one of the base stations/access points over the others.
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If a user experiences equally good channel conditions to several base stations/access points, the user forms a beam that favors one of the base stations/access points over the others.

If the power margin is sufficient without beam forming, the user can use the two antennas for spatial multiplexing.
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Pro: A sufficient power margin can be established (with high probability).

Con: Users at cell boundaries may suffer from reduced data rate.
Conclusions

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- Pilot contamination is not a fundamental effect, but an artefact of linear channel estimation.
- The algorithm requires real-time eigenvalue or singular value decompositions.


