Optimal linear receivers for large-scale multiuser MIMO systems

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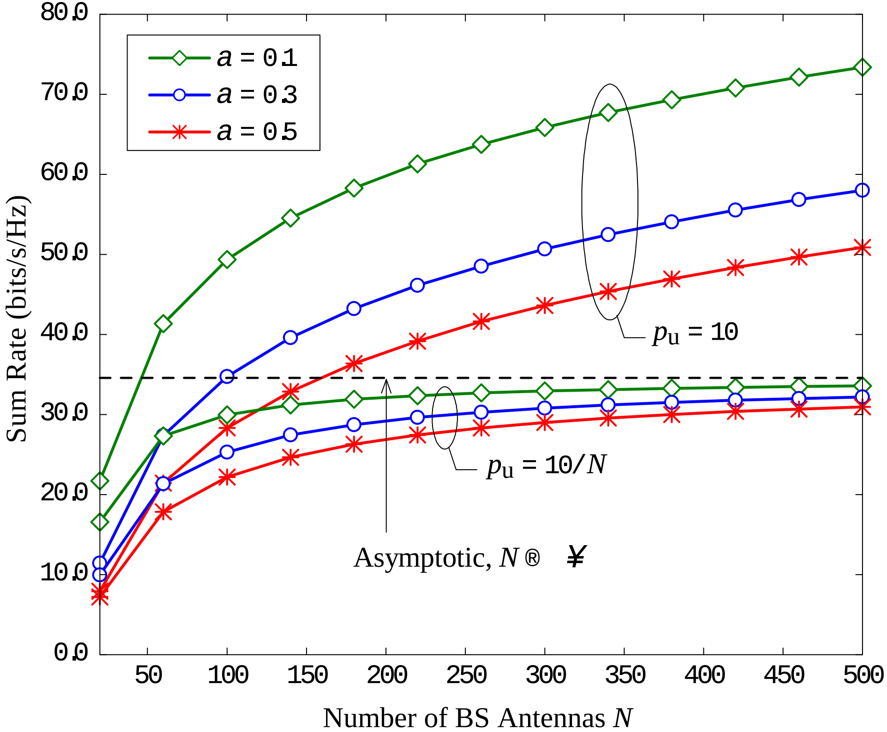
What advantages large MIMO systems offer?

- Dramatic capacity gains with very simple linear receivers (e.g. Maximum-Ratio combining, Zero-forcing)
- Power savings
- Random impairments (e.g., thermal noise and intercell interference) can be averaged out
- Increased reliability (in case one antenna fails, the system performance is not dramatically degraded)
- With large MIMO, expensive 40-Watt ultra-linear amplifiers are replaced by many cheap low-power devices whose combined action, only, has to meet stipulated tolerances
- Several expensive and bulky items, such as large coaxial cables, can be eliminated altogether (The coaxial cables used for tower mounted base stations today are up to four centimeters in diameter!)

- Multicell multiuser MIMO system with 4 cells and 10 users in each cell. All BS operate in the same frequency band. Uplink transmission and ZF reception at the BS.



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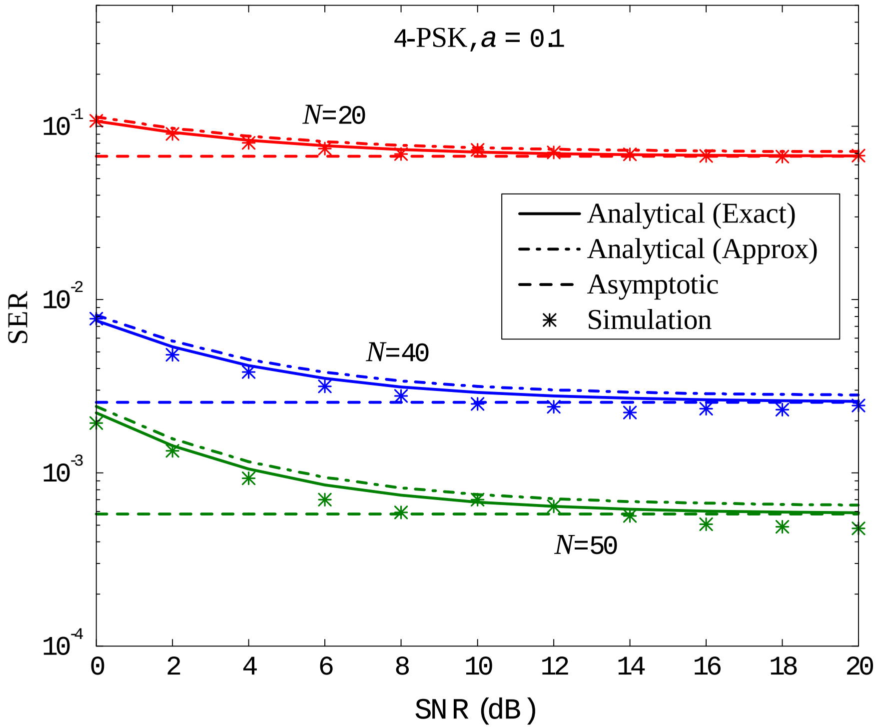

Research challenges

- Scalable processing (e.g. channel estimation with a very high number of coefficients)
- Realistic channel models & experimental research in large MIMO propagation
- Antenna design (mutual coupling suppression, 1D or 2D?)
- CSI acquisition of $N \cdot K$ channels $\rightarrow$ fundamental tradeoff between the time spent collecting CSI, and the time allocated to sending beamformed data to those users
- **Pilot contamination**: Due to the limitations of the channel coherence time, in multicell MU-MIMO systems with very large antenna arrays, non-orthogonal pilot sequences must be utilized in different cells. Therefore, the channel estimate in a given cell is impaired by the pilots transmitted by users in other cells!

System model

- Consider a multicell multiuser-MIMO system with L cells. Each cell includes one BS equipped with N antennas, and K single-antenna users $(N > K)$. We consider uplink transmission, and assume that the L BSs share the same frequency band. The $N \times 1$ received vector at the l-th BS is given by

$$\mathbf{y}_l = \sqrt{p_\mathrm{u}} \sum_{i=1}^{L} \mathbf{G}_{li} \mathbf{x}_i + \mathbf{n}_l \tag{1}$$

- where $\mathbf{G}_{li}$ is the $N \times K$ channel matrix between the l-th base station and the K users in the i-th cell, i.e., $g_{limk} \triangleq [\mathbf{G}_{li}]_{mk}$ is the channel coefficient between the m-th antenna of the l-th base station and the k-th user in the i-th cell; $\sqrt{p_\mathrm{u}}\mathbf{x}_i$ is the $K \times 1$ transmitted vector of K users in the i-th cell (the average power used by each user is p_u); and $\mathbf{n}_l$ is an $N \times 1$ additive white Gaussian noise (AWGN) vector

Channel estimation

- MMSE channel estimation with an interval of length τ for uplink training

$$\hat{\mathbf{g}}_{lik} = \beta_{lik} \left(\sum_{j=1}^{L} \beta_{ljk} + \frac{1}{\tau p_{\mathrm{p}}} \right)^{-1} \left(\sum_{j=1}^{L} \mathbf{g}_{ljk} + \frac{1}{\sqrt{\tau p_{\mathrm{p}}}} \mathbf{w}_{lk} \right) \quad (2)$$

where $\mathbf{w}_{lk} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_N\right)$, and p_{p} is the transmit power of each pilot symbol.

Channel estimation

- MMSE channel estimation with an interval of length τ for uplink training
- We can now decompose $\mathbf{g}_{lik}$ as

$$\mathbf{g}_{lik} = \hat{\mathbf{g}}_{lik} + \boldsymbol{\varepsilon}_{lik}$$

where $\boldsymbol{\varepsilon}_{llk}$ is the estimation error. We have

$$\hat{\mathbf{g}}_{lik} \sim \mathcal{CN}\left(\mathbf{0}, \frac{\beta_{lik}^2}{\sum_{j=1}^{L} \beta_{ljk} + 1/(\tau p_{\mathrm{p}})} \mathbf{I}_M\right)$$

$$\boldsymbol{\varepsilon}_{lik} \sim \mathcal{CN}\left(\mathbf{0}, \beta_{lik}\left(1 - \frac{\beta_{lik}}{\sum_{j=1}^{L} \beta_{ljk} + 1/(\tau p_{\mathrm{p}})}\right) \mathbf{I}_M\right).$$

Note that since we use MMSE channel estimation scheme, the channel estimate $\hat{\mathbf{g}}_{lik}$ is independent of the estimation error $\boldsymbol{\varepsilon}_{lik}$!

Linear receivers

- Linear receivers: Let $\mathbf{A}_l$ be an $N \times K$ linear detection matrix which depends on the channel estimates $\hat{\mathbf{G}}_{li}$, $i = 1, ..., L$. The lth BS processes its received signal by multiplying it by $\mathbf{A}_l^H$ as follows

$$\mathbf{r}_l = \mathbf{A}_l^H \mathbf{y}_l = \sqrt{p_{\mathrm{u}}} \mathbf{A}_l^H \sum_{i=1}^{L} \mathbf{G}_{li} \mathbf{x}_i + \mathbf{A}_l^H \mathbf{n}_l. \quad (3)$$

Linear receivers

- Let $\mathbf{a}_{lk}$ be the kth column of $\mathbf{A}_l$ and $\boldsymbol{\mathcal{E}}_{li} \triangleq [\boldsymbol{\varepsilon}_{li1} \ldots \boldsymbol{\varepsilon}_{liK}]$. Then, the kth element of $\mathbf{r}_l$ is

$$r_{lk} = \underbrace{\sqrt{p_{\rm u}}\mathbf{a}_{lk}^H \hat{\mathbf{g}}_{llk} x_{lk}}_{\text{desired signal}} + \underbrace{\sqrt{p_{\rm u}} \sum_{j=1, j\neq k}^{K} \mathbf{a}_{lk}^H \hat{\mathbf{g}}_{llj} x_{lj}}_{\text{intracell interference}}$$
$$+ \underbrace{\sqrt{p_{\rm u}}\mathbf{a}_{lk}^H \sum_{i=1, i\neq l}^{L} \hat{\mathbf{G}}_{li} \mathbf{x}_i}_{\text{intercell interference}} + \underbrace{\sqrt{p_{\rm u}}\mathbf{a}_{lk}^H \sum_{i=1l}^{L} \boldsymbol{\mathcal{E}}_{li} \mathbf{x}_i}_{\text{estimate-error interference}} + \underbrace{\mathbf{a}_{lk}^H \mathbf{n}_l}_{\text{noise}} \quad (4)$$

- Since $\hat{\mathbf{G}}_{li} = \hat{\mathbf{G}}_{ll}\mathbf{D}_i$, where $\mathbf{D}_i = \mathrm{diag}\left\{\frac{\beta_{li1}}{\beta_{ll1}}, \frac{\beta_{li2}}{\beta_{ll2}}, \ldots, \frac{\beta_{liK}}{\beta_{llK}}\right\}$, we can infer that intercell interference is correlated with the channel estimate $\hat{\mathbf{G}}_{ll}$!

Linear receivers

- The SINR of the uplink transmission from the kth user in lth cell to its BS is given by

$$\mathtt{SINR}_{\mathrm{k}} = \frac{p_{\mathrm{u}} \left|\mathbf{a}_{lk}^{H}\hat{\mathbf{g}}_{llk}\right|^2}{p_{\mathrm{u}} \sum_{j\neq k}^{K} \left|\mathbf{a}_{lk}^{H}\hat{\mathbf{g}}_{llj}\right|^2 + p_{\mathrm{u}} \sum_{i\neq l}^{L} \left\|\mathbf{a}_{lk}^{H}\hat{\mathbf{G}}_{li}\right\|^2 + p_{\mathrm{u}} \sum_{i=1}^{L} \gamma_{li}\|\mathbf{a}_{lk}\|^2 + \|\mathbf{a}_{lk}\|^2} \tag{5}$$

where

$$\gamma_{li} \triangleq \sum_{k=1}^{K} \left[\beta_{lik} - \beta_{lik}^2 \left(\sum_{j=1}^{L} \beta_{ljk} + \frac{1}{\tau p_{\mathrm{p}}}\right)^{-1}\right]. \tag{6}$$

Linear receivers

- MMSE is the optimal linear receiver for single-cell systems
- What is the optimal linear receiver (OLR) for multi-cell systems?
- In multi-cell systems, channel estimate is contaminated by pilots transmitted from other cells. Thus, channel estimate and the interference from other cells are correlated

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- Conventional MMSE receivers: power(desired signal)/power(interference)
- Conventional MMSE receivers: intercell interference is treated as uncorrelated noise such that

$$\mathbf{a}_{lk} = \left(\hat{\mathbf{G}}_{ll} \hat{\mathbf{G}}_{ll}^H + \mathrm{E}\left\{ \boldsymbol{\mathcal{E}}_{ll} \boldsymbol{\mathcal{E}}_{ll}^H \right\} + \sum_{i \neq l}^{L} \mathrm{E}\left\{ \mathbf{G}_{li} \mathbf{G}_{li}^H \right\} + \frac{1}{p_{\mathrm{u}}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{g}}_{llk}. \qquad (7)$$

Optimal linear receivers

- It can be shown that the SINR is equal to

$$\mathtt{SINR}_{\mathrm{k}} = \frac{\left|\mathbf{a}_{lk}^{H}\hat{\mathbf{g}}_{llk}\right|^2}{\mathbf{a}_{lk}^{H}\boldsymbol{\Xi}_k\mathbf{a}_{lk}} \tag{8}$$

where

$$\boldsymbol{\Xi}_k \triangleq \sum_{j=1,\ j\neq k}^{K} \hat{\mathbf{g}}_{llj}\hat{\mathbf{g}}_{llj}^{H} + \sum_{i=1,\ i\neq l}^{L} \hat{\mathbf{G}}_{li}\hat{\mathbf{G}}_{li}^{H} + \left(\sum_{i=1}^{L}\gamma_{li} + \frac{1}{p_{\mathrm{u}}}\right)\mathbf{I}_M .$$

- OLR: power(desired signal)/power(interference for a given $\hat{\mathbf{G}}_{ll}$)

Optimal linear receivers

- Using Cauchy-Schwarz's inequality, we have

$$\mathtt{SINR_k} = \frac{\left|\mathbf{a}_{lk}^H \hat{\mathbf{g}}_{llk}\right|^2}{\mathbf{a}_{lk}^H \boldsymbol{\Xi}_k \mathbf{a}_{lk}} = \frac{\left|\mathbf{a}_{lk}^H \boldsymbol{\Xi}_k^{1/2} \boldsymbol{\Xi}_k^{-1/2} \hat{\mathbf{g}}_{llk}\right|^2}{\mathbf{a}_{lk}^H \boldsymbol{\Xi}_k \mathbf{a}_{lk}}$$
$$\leq \frac{\left\|\mathbf{a}_{lk}^H \boldsymbol{\Xi}_k^{1/2}\right\|^2 \left\|\boldsymbol{\Xi}_k^{-1/2} \hat{\mathbf{g}}_{llk}\right\|^2}{\mathbf{a}_{lk}^H \boldsymbol{\Xi}_k \mathbf{a}_{lk}} = \left\|\boldsymbol{\Xi}_k^{-1/2} \hat{\mathbf{g}}_{llk}\right\|^2. \tag{9}$$

- The equality holds when $\mathbf{a}_{lk} = c\boldsymbol{\Xi}_k^{-1}\hat{\mathbf{g}}_{llk}$, for any $c \neq 0 \in \mathbb{C}$. Therefore, the optimal linear receiver at the lth BS is given by

$$\mathbf{a}_{lk} = c\boldsymbol{\Xi}_k^{-1}\hat{\mathbf{g}}_{llk}. \tag{10}$$

Large N analysis

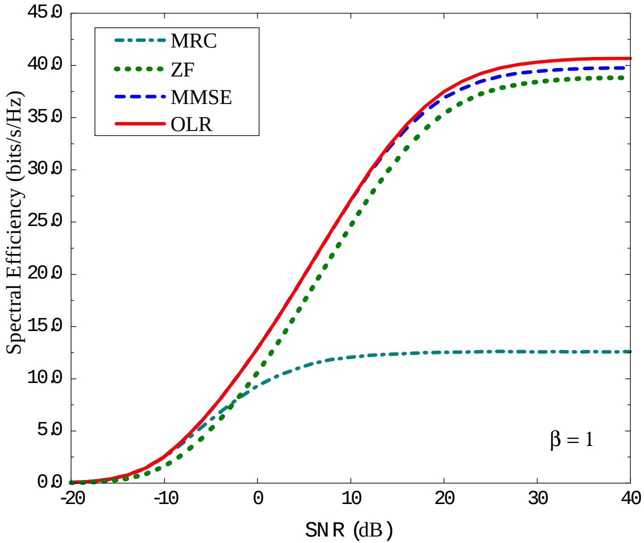
- We assume $p_{\rm p} = p_{\rm u}$. With $p_{\rm u} = E_{\rm u}/\sqrt{N}$, when N grows large, we have

$$\mathtt{SINR}_{\text{opt},k} \xrightarrow{a.s.} \frac{1}{1 - [\mathbf{D}^{-1}]_{kk} + [\mathbf{D}^{-1}]_{kk}\left(\tau E_{\rm u}^2 \sum_{i=1}^{L} \beta_{lik}^2 + 1\right)^{-1}} - 1$$
$$= \frac{\tau E_{\rm u}^2 \beta_{llk}^2}{\tau E_{\rm u}^2 \sum_{i=1, i\neq l}^{L} \beta_{lik}^2 + 1} \quad (11)$$

- With OLR and very large antenna arrays at the BS, we can reduce the transmit power proportionally to $1/\sqrt{N}$ while maintaining a desired QoS. This result coincides with the asymptotic results of MRC, ZF, and MMSE!

Optimal linear receivers

- In conventional MMSE receivers, the intercell interference is treated as noise. The proposed OLR takes the correlation between the channel estimate and the intercell interference into account ($L = 7$ cells, $N = 20$, $K = 10$ users/cell, coherence interval $T = 196$ symbols and $\tau = K = 10$ and $p_{\mathrm{p}} = p_{\mathrm{u}}$).



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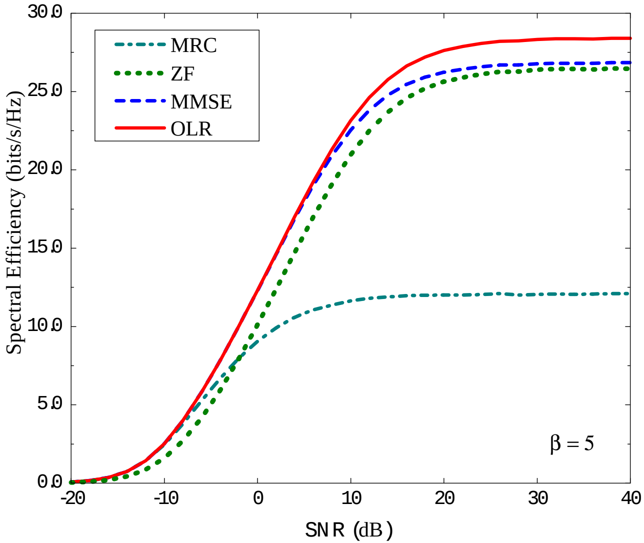
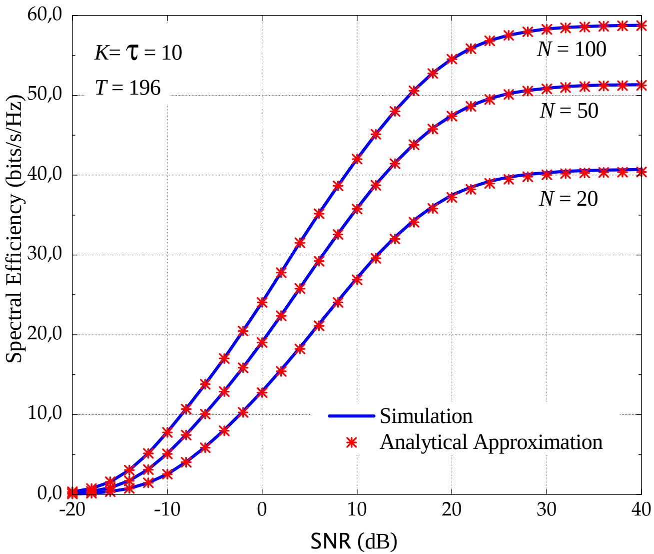


Approximating lower bound for OLRs

- The lower bound on the achievable rate of the kth user in the lth cell is approximated as

$$\tilde{R}_{\text{opt},k} \approx -\log_2\left(1 - \left[\mathbf{D}^{-1}\right]_{kk} + \left[\mathbf{D}^{-1}\right]_{kk}\, {}_2F_0\left(\alpha_k, 1; —; -\theta_k\right)\right) \quad (12)$$



Conclusions

- Large MIMO is a novel and highly promising technology for improving the performance of concurrent communication systems and meeting the future mobile data traffic demands
- Linear receivers are of particular importance for large-scale MIMO systems, due to their low complexity and near-optimal performance
- The optimal linear receiver for single-cell systems is MMSE
- The optimal linear receiver for multi-cell systems is unknown
- By taking the correlation between the channel estimate and the intercell interference into account, we can derive the OLR
- The OLR outperforms all known linear receivers (e.g. MRC, ZF, MMSE) especially in strong interference conditions.

Material presented in this talk is taken from the following papers:

1. H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Large-scale MU-MIMO: Optimal linear receivers and resource allocation," submitted to *IEEE Transactions on Signal Processing*, March 2013.

2. H. Q. Ngo, M. Matthaiou, T. Q. Duong, and E. G. Larsson, "Uplink performance analysis of multicell MU-SIMO systems with ZF receivers," to appear *IEEE Transactions on Vehicular Technology*, 2013.

3. H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Performance analysis of large scale MU-MIMO with optimal linear receivers," in *Proc. IEEE Swedish Communication Technologies Workshop (Swe-CTW)*, Lund, Sweden, October 2012, pp. 59–64.

Thank you!