Coverage and Capacity Analysis of mmWave Cellular Systems

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**Why mmWave for Cellular?**

<table>
<thead>
<tr>
<th>1G-4G cellular</th>
<th>5G cellular</th>
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<tbody>
<tr>
<td><strong>Microwave</strong></td>
<td><strong>mmWave</strong></td>
</tr>
<tr>
<td>300 MHz</td>
<td>300 GHz</td>
</tr>
<tr>
<td>3 GHz</td>
<td></td>
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<tr>
<td>28 GHz</td>
<td></td>
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<tr>
<td>38-49 GHz</td>
<td></td>
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<tr>
<td>70-90 GHz</td>
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</table>

- **Huge amount of spectrum available in mmWave bands***
  - Cellular systems live with limited microwave spectrum ~ 600MHz
  - 29GHz **possibly** available in 23GHz, LMDS, 38, 40, 46, 47, 49, and E-band

- **Technology advances make mmWave possible**
  - Silicon-based technology enables low-cost highly-packed mmWave RFIC**
  - Hybrid precoding solves RF limitations
  - Commercial products already available (or soon) for PAN and LAN

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**The Need for Gain**

- **Smaller wavelength** means **smaller captured energy** at antenna
  - 3GHz→30GHz gives 20dB extra path loss due to aperture
- **Larger bandwidth** means **higher noise power** and lower SNR
  - 50MHz → 500MHz bandwidth gives 10dB extra noise power

Solution: Exploit array gain from large antenna arrays
Antenna Arrays are Important

Narrow beams are a new feature of mmWave
- Reduces fading, multi-path, and interference
- Implemented in analog due to hardware constraints

Arrays will change system design principles

Different Propagation Behavior

- Buildings block Line-of-sight (LOS) paths
  - mmWave signal suffer from high penetration losses
- Reflections can establish non-LOS links
  - Best non-LOS links still tens of dB weaker than LOS signals
- Different characteristics for LOS & non-LOS
  - Path loss exponent is around 2 for LOS, and 4 for non-LOS

Challenges of mmWave Analysis

Need to incorporate directional beamforming
- RX and TX communicate via main lobes to achieve array gain
- Steering directions at interfering BSs are random

Need to distinguish LOS and non-LOS paths
- Dramatically different characteristics in LOS & non-LOS channels
- Better characterize blockages

How to including beamforming + blockages in mmWave cellular analysis?
Stochastic Geometry for mmWave Cellular System Analysis
Stochastic Geometry for Cellular

Stochastic geometry is a tool for analyzing microwave cellular

- Reasonable fit with real deployments
- Closed form solutions for **coverage probability** available
- Provides a **system-wide performance characterization**

Need to incorporate **LOS/non-LOS links** and **directional antennas**

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Poisson point process (PPP): the simplest point process

- # of points is a Poisson variable with mean $\lambda S$
- Given $N$ points in certain area, locations independent
- Useful results like Campbell’s Theorem & Displacement Theorem apply
- Assigning each point an i.i.d. random variable forms a marked PPP

Antenna steering orientations as marks of the BS PPP
Blockages in mmWave

Use random shape theory to model buildings

- Model random buildings as rectangle Boolean scheme
- Buildings distributed as PPP with independent sizes & orientations

Compute the LOS probability based on the building model

- # of blockages on a link is a Poisson random variable
- The LOS probability that no blockage on a link of length $R$ is $e^{-\beta R}$

Directional Transmission at the BS

- Each base station is marked with a directional antenna
- Antenna directions of interferers are uniformly distributed
- Use “sector” pattern in analysis for simplicity
  - Equivalent to uniform linear array of $N_t$ antennas with spacing $\lambda/2$

**Main lobe beamwidth:**
$$\theta = 2 \arcsin \left( \frac{2.782}{\pi N_t} \right)$$

**Main lobe array gain:**
$$M = N_t \quad \# \text{antennas}$$

**Front-back ratio:**
$$FBR = \sin \left( \frac{3\pi}{2N_t} \right)$$

**Back lobe gain:**
$$m = N_t \times FBR$$
Proposed mmWave Model

- Use stochastic geometry to model BSs as marked PPP
  - Model the steering directions as independent marks of the BSs
- Use random shape theory to model buildings
  - Model the building as rectangle Boolean schemes
  - Different path loss exponents for LOS and non-LOS paths
System Parameters

- Different path loss model for LOS and nonLOS links
  - Line-of-sight with probability $e^{-\beta R}$
  - LOS path Loss in dB: $PL_1 = C + 20 \log R(m)$
  - Non-LOS path loss in dB: $PL_2 = C + K + 40 \log R(m)$
  - 28Ghz system: let $C=50$ dB, $K=10$ dB

- General small scale fading $h$
  - No fading case: small scaling fading is minor in mmWave [RapSun]

- Link budget
  - Tx antenna input power: 30dBm
  - Signal bandwidth: 500 MHz (Noise: -87 dBm)
  - Noise figure: 5dB
Results on Coverage
Coverage Analysis

\[
\text{SINR} = \frac{N_r N_t H_0 \ell(r_0)}{N_0 / P_t + \sum_{k>0} A_k B_k H_k \ell(r_k)},
\]

where

\[
H_0 \ell(r_0) = \min_{k>0} \{H_k \ell(r_k)\},
\]

\[
A_k = \begin{cases} 
N_t & \text{w. p. } \frac{\theta_t}{2\pi}, \\
N_t FBR_t & \text{w. p. } 1 - \frac{\theta_t}{2\pi}, 
\end{cases}
\]

\[
B_k = \begin{cases} 
N_r & \text{w. p. } \frac{\theta_r}{2\pi}, \\
N_r FBR_r & \text{w. p. } 1 - \frac{\theta_r}{2\pi}, 
\end{cases}
\]

\[
\ell(x) = \begin{cases} 
Cx^{-2} & \text{w. p. } e^{-\beta x}, \\
C' K x^{-4} & \text{w. p. } 1 - e^{-\beta x}.
\end{cases}
\]

Serving BS and User connect via main lobe

General small-scale fading

Connecting to the strongest signal before BF

Array gain of the TX antenna

Array gain of the RX antenna

Path Loss of LOS or non-LOS

Use stochastic geometry to compute SINR distribution
Coverage Probability of mmWave

Main Theorem [mmWave Coverage probability]

The coverage probability $\mathbb{P}[\text{SINR} > T]$ can be computed as

$$
\mathbb{P}(\text{SINR} > T) = \int_{-\infty}^{\infty} \int_{0}^{\infty} U(x, t) f_{L^*}(x) \frac{e^{j2\pi t/T} - 1}{j2\pi t} dx dt
$$

where

$$
U(x, s) = \exp \left[ -\frac{sx}{M_t M_r \rho} + \int_{x}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{s x M_t}{u M_r}} + p_t (1 - p_r) e^{-\frac{s x M_t M_r}{u M_t M_r}} - 1 \right) \Lambda(du) \right],
$$

$$
\Lambda(x) = 2\pi \lambda \mathbb{E}_h \left[ \int_{0}^{\sqrt{xh}} t (1 - e^{-\beta t}) dt + \int_{\sqrt{xh}}^{\infty} t e^{-\beta t} dt \right],
$$

$$
f_{L^*}(x) = -\frac{d}{dx} e^{-\Lambda(x)}.
$$

Transform interference field into 1D space by Displacement Thm
Coverage Gain from Large Arrays

Mobile user 16 antennas
BS density Rc=200 m
Buildings cover 5% of the land
Average building size 15 m by 15 m

Large arrays provide better coverage probability
- the more antennas, the smaller beamwidth, the larger array gain
- Smaller beamwidth provides better coverage
- mmWave coverage probability comparable to microwave

Gain from directional antenna array

- $N_t=64$, $\theta=1.6^\circ$
- $N_t=32$, $\theta=3.2^\circ$
- $N_t=16$, $\theta=6.5^\circ$
- Microwave: SU MIMO 4X4
Higher density can also increase coverage probability

- Coverage probability no longer invariant with BS density
- Become interference-limited when coverage probability is good
Coverage probability differs in LOS and non-LOS region

- Need to incorporate blockage model & differentiate LOS and non-LOS
- Non-LOS coverage probability generally provides a lower bound
- Buildings may improve coverage by blocking more interference
Corollary 1 [Coverage probability by LOS BSs]

The coverage probability provided by LOS BSs is

\[
\mathbb{P}(\text{SINR} > T) = \int_{-\infty}^{\infty} \int_{0}^{\infty} U_1(x, t) U_2(t) f_{L^*}(x) \frac{e^{j2\pi t/T} - 1}{j2\pi t} dx dt,
\]

where

\[
U_1(x, s) = \exp \left[ -\frac{sx}{M_t M_\rho} + \int_{x}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{s\tilde{x}M_t}{uM_t}} + p_t (1 - p_r) e^{-\frac{s\tilde{x}M_t}{uM_t}} \right) \Lambda_1(du) \right],
\]

\[
U_2(s) = \exp \left[ \int_{0}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{s\tilde{x}M_t}{uM_t}} + p_t (1 - p_r) e^{-\frac{s\tilde{x}M_t}{uM_t}} \right) \Lambda_2(du) \right],
\]

\[
\Lambda_1(x) = 2\pi \lambda \int_{0}^{\sqrt{x}} t e^{-\beta t} dt,
\]

\[
\Lambda_2(x) = 2\pi \lambda \int_{0}^{(\frac{x}{K})^{1/4}} t (1 - e^{-\beta t}) dt,
\]

\[
f_{L^*}(x) = -\frac{d}{dx} e^{-\Lambda_1(x)}.
\]

Coverage probability by reflections derived in a similar way
Reflections Improve Coverage

128 antennas at BSs
Blockages covers 30% land
(Heavy shadowing case)
Rc=200 m

Reflections can establish links in the shadowed areas
- With dense blockages, most users are served by reflected links
- Non-LOS links improve the coverage probability of mmWave

With dense blockages, a large fraction of users is non-LOS

Coverage Probability
SINR Threshold

Overall coverage probability
Coverage probability by LOS BSs
Coverage probability by reflections
Results on Rate
Data Rate Comparison

Given coverage probability, the achievable rate is
\[
\eta = \int_0^\infty \frac{P_c(T)}{1 + T} dT
\]

Microwave network 4X4 MU MIMO with bandwidth 50MHz:
- Spectrum efficiency is 4.95 bps/ Hz
- Data rate is 248 Mbps (invariant with the cell size Rc)

mmWave network with bandwidth 500MHz:

<table>
<thead>
<tr>
<th>( N_t )</th>
<th>( R_c )</th>
<th>100m</th>
<th>200m</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>100m</td>
<td>3.4Gbps</td>
<td>3.25Gbps</td>
</tr>
<tr>
<td>32</td>
<td>200m</td>
<td>3.25Gbps</td>
<td>3.1Gbps</td>
</tr>
<tr>
<td>64</td>
<td>100m</td>
<td>3.8Gbps</td>
<td>3.45Gbps</td>
</tr>
<tr>
<td>64</td>
<td>200m</td>
<td>3.45Gbps</td>
<td>3.25Gbps</td>
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mmWave achieves high gain in average rate
Cell Throughput Comparison

Gain from larger bandwidth

Gain from serving multiple users

mmWave can support much higher data rate

Conclusions
Going Forward with mmWave

- mmWave coverage probability and rate
  - Need to include both LOS and Non-LOS conditions
  - Interference is reduced by directional antennas and blockages
  - Good rates and coverage can be achieved

- Theoretical challenges abound
  - Analog beamforming algorithms & hybrid beamforming
  - Channel estimation, exploiting sparsity, incorporating robustness
  - Multi-user beamforming algorithms and analysis
  - Microwave-overlaid mmWave system a.k.a. phantom cells
  - More advanced stochastic geometry models including multi-tier
Questions?

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See forthcoming book:

Millimeter Wave Wireless Communication

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